Množinový kalkul

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Nauč sa matiku

Obsah

Ak je objekt x *prvkom* množiny A, píšeme $x \in A$, inak $x \notin A$.

- $=$ ___ $\{+, -, \cdot, \div, =, (,)\}$
- Star Wars ___ Sci-fi filmy
- a ___ $\{\{a\}, \{b, c\}\}$
- $\{a\}$ ___ $\{\{a\}, \{b, c\}\}$
- 0 ___ $\emptyset$
- $\emptyset$ ___ $\emptyset$
- $\emptyset$ ___ $\{\emptyset\}$
- $\emptyset$ ___ $\{\{\emptyset\}\}$

Vzťah „byť prvkom"

Ak je objekt x *prvkom* množiny A, píšeme $x \in A$, inak $x \notin A$.

- $= \in \{+, -, \cdot, \div, =, (,)\}$
- Star Wars $\in$ Sci-fi filmy
- $a \notin \{\{a\}, \{b, c\}\}$
- $\{a\} \in \{\{a\}, \{b, c\}\}$
- $0 \notin \emptyset$
- $\emptyset \notin \emptyset$
- $\emptyset \in \{\emptyset\}$
- $\emptyset \notin \{\{\emptyset\}\}$

$A \subseteq B \iff^{\text{def.}} \forall x \colon (x \in A \implies x \in B)$.

- Eva ___ Ženské mená
- $\{1, 2\}$ ___ $\{1, 2, 3\}$
- $\{1, 2\}$ ___ $\{1, 2\}$
- $\emptyset$ ___ $\{1, 2, 3\}$
- $\emptyset$ ___ $\emptyset$

Vzťah inklúzie (podmnožiny)

$A \subseteq B \overset{def.}{\iff} \forall x\colon (x \in A \implies x \in B)$.

- Eva $\not\subseteq$ Ženské mená
- $\{1, 2\} \subseteq \{1, 2, 3\}$
- $\{1, 2\} \subseteq \{1, 2\}$
- $\emptyset \subseteq \{1, 2, 3\}$
- $\emptyset \subseteq \emptyset$

Vzťah rovnosti

$A = B \overset{def.}{\iff} \forall x\colon (x \in A \iff x \in B).$

- $\{1, 2, 3\}$ ___ $\{3, 1, 2\}$
- $\{1, 2, 3\}$ ___ $\{1, 2, 1, 2, 3, 3, 3, 3\}$
- $\emptyset$ ___ $\emptyset$
- $\emptyset$ ___ $\{\emptyset\}$

Vzťah rovnosti

$$A = B \stackrel{def.}{\iff} \forall x \colon (x \in A \iff x \in B).$$

- $\{1, 2, 3\} = \{3, 1, 2\}$
- $\{1, 2, 3\} = \{1, 2, 1, 2, 3, 3, 3, 3\}$
- $\emptyset = \emptyset$
- $\emptyset \neq \{\emptyset\}$

Vzťah rovnosti

$A = B \iff \forall x : (x \in A \iff x \in B).$

- $\{1, 2, 3\} = \{3, 1, 2\}$
- $\{1, 2, 3\} = \{1, 2, 1, 2, 3, 3, 3, 3\}$
- $\emptyset = \emptyset$
- $\emptyset \neq \{\emptyset\}$

Tiež $A = B \iff (A \subseteq B) \wedge (B \subseteq A).$

Vzťah rovnosti

$A = B \iff \forall x \colon (x \in A \iff x \in B).$

- $\{1, 2, 3\} = \{3, 1, 2\}$
- $\{1, 2, 3\} = \{1, 2, 1, 2, 3, 3, 3, 3\}$
- $\emptyset = \emptyset$
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Tiež $A = B \iff (A \subseteq B) \wedge (B \subseteq A).$

Podmnožina: $A \subseteq B \iff \forall x \colon (x \in A \implies x \in B).$

Vzťah rovnosti

$A = B \iff^{\text{def.}} \forall x \colon (x \in A \iff x \in B).$

- $\{1, 2, 3\} = \{3, 1, 2\}$
- $\{1, 2, 3\} = \{1, 2, 1, 2, 3, 3, 3, 3\}$
- $\emptyset = \emptyset$
- $\emptyset \neq \{\emptyset\}$

Tiež $A = B \iff^{\text{def.}} (A \subseteq B) \wedge (B \subseteq A).$

Podmnožina: $A \subseteq B \iff^{\text{def.}} \forall x \colon (x \in A \implies x \in B).$
Vlastná podmnožina: $A \subset B \iff^{\text{def.}} A \subseteq B \wedge A \neq B.$

Vzťah rovnosti

$A = B \iff^{\text{def.}} \forall x \colon (x \in A \iff x \in B).$

- $\{1, 2, 3\} = \{3, 1, 2\}$
- $\{1, 2, 3\} = \{1, 2, 1, 2, 3, 3, 3, 3\}$
- $\emptyset = \emptyset$
- $\emptyset \neq \{\emptyset\}$

Tiež $A = B \iff^{\text{def.}} (A \subseteq B) \wedge (B \subseteq A).$

Podmnožina: $A \subseteq B \iff^{\text{def.}} \forall x \colon (x \in A \implies x \in B).$
Vlastná podmnožina: $A \subset B \iff^{\text{def.}} A \subseteq B \wedge A \neq B.$

- $\emptyset \underline{} \emptyset$

Vzťah rovnosti

$A = B \xrightarrow{def.} \forall x : (x \in A \iff x \in B).$

- $\{1, 2, 3\} = \{3, 1, 2\}$
- $\{1, 2, 3\} = \{1, 2, 1, 2, 3, 3, 3, 3\}$
- $\emptyset = \emptyset$
- $\emptyset \neq \{\emptyset\}$

Tiež $A = B \xrightarrow{def.} (A \subseteq B) \wedge (B \subseteq A).$

Podmnožina: $A \subseteq B \xrightarrow{def.} \forall x : (x \in A \implies x \in B).$
Vlastná podmnožina: $A \subset B \xrightarrow{def.} A \subseteq B \wedge A \neq B.$

- $\emptyset \not\subset \emptyset$

Operácia zjednotenia

$A \cup B \overset{\text{def.}}{=} \{x \mid x \in A \lor x \in B\}$.

- $\{\text{Adam}\} \cup \{\text{Eva}\} = \underline{}$
- $\{c, b, a\} \cup \{c, b, d\} = \underline{}$
- $\{\{x\}\} \cup \{x\} = \underline{}$
- $\{c, b, a\} \cup \emptyset = \underline{}$
- $\emptyset \cup \emptyset = \underline{}$

$A \cup B \stackrel{\text{def.}}{=} \{x \mid x \in A \lor x \in B\}$.

- $\{\text{Adam}\} \cup \{\text{Eva}\} = \{\text{Adam, Eva}\}$
- $\{c, b, a\} \cup \{c, b, d\} = \{a, b, c, d\}$
- $\{\{x\}\} \cup \{x\} = \{x, \{x\}\}$
- $\{c, b, a\} \cup \emptyset = \{a, b, c\}$
- $\emptyset \cup \emptyset = \emptyset$

Operácia prieniku

$A \cap B \overset{\text{def.}}{=} \{x \mid x \in A \land x \in B\}.$

- $\{\text{Adam}\} \cap \{\text{Eva}\} = \underline{}$
- $\{c, b, a\} \cap \{c, b, d\} = \underline{}$
- $\{\{x\}\} \cap \{x\} = \underline{}$
- $\{c, b, a\} \cap \emptyset = \underline{}$
- $\emptyset \cap \emptyset = \underline{}$

Operácia prieniku

$A \cap B \overset{\text{def.}}{=} \{x \mid x \in A \land x \in B\}$.

- $\{\text{Adam}\} \cap \{\text{Eva}\} = \emptyset$
- $\{c, b, a\} \cap \{c, b, d\} = \{b, c\}$
- $\{\{x\}\} \cap \{x\} = \emptyset$
- $\{c, b, a\} \cap \emptyset = \emptyset$
- $\emptyset \cap \emptyset = \emptyset$

Operácia rozdielu

$A \setminus B \stackrel{def.}{=} \{x \mid x \in A \wedge x \notin B\}.$

- $\{\text{Adam}\} \setminus \{\text{Eva}\} = $ ___
- $\{c, b, a\} \setminus \{c, b, d\} = $ ___
- $\{\{x\}\} \setminus \{x\} = $ ___
- $\{c, b, a\} \setminus \emptyset = $ ___
- $\emptyset \setminus \emptyset = $ ___

Operácia rozdielu

$A \setminus B \stackrel{\text{def.}}{=} \{x \mid x \in A \land x \notin B\}.$

- $\{\text{Adam}\} \setminus \{\text{Eva}\} = \{\text{Adam}\}$
- $\{c, b, a\} \setminus \{c, b, d\} = \{a\}$
- $\{\{x\}\} \setminus \{x\} = \{\{x\}\}$
- $\{c, b, a\} \setminus \emptyset = \{c, b, a\}$
- $\emptyset \setminus \emptyset = \emptyset$

Operácia symetrického rozdielu

$A \triangle B \stackrel{def.}{=} (A \setminus B) \cup (B \setminus A)$.

- $\{\text{Adam}\} \triangle \{\text{Eva}\} = \underline{\quad}$
- $\{c, b, a\} \triangle \{c, b, d\} = \underline{\quad}$
- $\{\{x\}\} \triangle \{x\} = \underline{\quad}$
- $\{c, b, a\} \triangle \emptyset = \underline{\quad}$
- $\emptyset \triangle \emptyset = \underline{\quad}$
- $\{1, 5, 9\} \triangle \{1, 5, 9\} = \underline{\quad}$

Operácia symetrického rozdielu

$A \triangle B \overset{\text{def.}}{=} (A \setminus B) \cup (B \setminus A).$

- $\{\text{Adam}\} \triangle \{\text{Eva}\} = \{\text{Adam, Eva}\}$
- $\{c, b, a\} \triangle \{c, b, d\} = \{a, d\}$
- $\{\{x\}\} \triangle \{x\} = \{x, \{x\}\}$
- $\{c, b, a\} \triangle \emptyset = \{c, b, a\}$
- $\emptyset \triangle \emptyset = \emptyset$
- $\{1, 5, 9\} \triangle \{1, 5, 9\} = \emptyset$

Ak $A \subseteq U$, tak doplnok množiny A vzhľadom k U je množina $\overline{A} \stackrel{def.}{=} U \setminus A$.

- $U = \{\text{Adam, Barbora, Cyril, Dana}\}$, $A = \{\text{Adam, Cyril}\}$, $\overline{A} = $ ___
- $U = \{c, b, a\}$, $\overline{U} = $ ___
- $U = \{c, b, a\}$, $\overline{\emptyset} = $ ___
- $U = \{1, \{2\}, \{\{3\}\}\}$, $A = \{1\}$, $\overline{A} = $ ___

Operácia doplnku

Ak $A \subseteq U$, tak doplnok množiny A vzhľadom k U je množina $\overline{A} \stackrel{def.}{=} U \setminus A$.

- $U = \{$Adam, Barbora, Cyril, Dana$\}, A = \{$Adam, Cyril$\}, \overline{A} = \{$Barbora, Dana$\}$
- $U = \{c, b, a\}, \overline{U} = \emptyset$
- $U = \{c, b, a\}, \overline{\emptyset} = U$
- $U = \{1, \{2\}, \{\{3\}\}\}, A = \{1\}, \overline{A} = \{\{2\}, \{\{3\}\}\}$

$A \times B \stackrel{\text{def.}}{=} \{(a, b) \mid a \in A \land b \in B\}$.

(a, b) je usporiadaná dvojica definovaná množinou $\{\{a\}, \{a, b\}\}$.

- $A = \{1, 2, 3\}$, $M = \{a, b\}$, $A \times M = $ ___
- $A = \{1, 2\}$, $B = \emptyset$, $A \times B = $ ___
- $A = \{1, 2\}$, $B = \{\emptyset\}$, $A \times B = $ ___
- $A = \{x, y\}$, $A^2 = $ ___
- $A = \{x, y\}$, $A^0 = $ ___

Operácia kartézskeho súčinu

$A \times B \overset{\text{def.}}{=} \{(a, b) \mid a \in A \wedge b \in B\}$.

(a, b) je usporiadaná dvojica definovaná množinou $\{\{a\}, \{a, b\}\}$.

- $A = \{1, 2, 3\}$, $M = \{a, b\}$, $A \times M =$
  $\{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$
- $A = \{1, 2\}$, $B = \emptyset$, $A \times B = \emptyset$
- $A = \{1, 2\}$, $B = \{\emptyset\}$, $A \times B = \{(1, \emptyset), (2, \emptyset)\}$
- $A = \{x, y\}$, $A^2 = \{(x, x), (x, y), (y, x), (y, y)\}$
- $A = \{x, y\}$, $A^0 = \{\emptyset\}$

Potenčná množina

$2^A \stackrel{def.}{=} \{B \mid B \subseteq A\}.$

- $M = \{a, b\},\ 2^M = \ \underline{\quad}$
- $2^\emptyset = \ \underline{\quad}$

Potenčná množina

$2^A \stackrel{\text{def.}}{=} \{B \mid B \subseteq A\}.$

- $M = \{a, b\},\ 2^M = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
- $2^\emptyset = \{\emptyset\}$

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Nauč sa matiku

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