

# Množinový kalkul

Valdemar Švábenský

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## ① Vzťahy medzi množinami

Vzťah „byť prvkom“

Inklúzia

Rovnosť

## ② Množinové operácie

Zjednotenie

Prienik

Rozdiel

Symetrický rozdiel

Doplňok

Kartézsky súčin

Potenčná množina

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## Vzťah „byť prvkom“

Ak je objekt  $x$  prvkom množiny  $A$ , píšeme  $x \in A$ , inak  $x \notin A$ .

- $=$  \_\_\_  $\{+, -, \cdot, \div, =, (, )\}$
- Star Wars \_\_\_ Sci-fi filmy
- $a$  \_\_\_  $\{\{a\}, \{b, c\}\}$
- $\{a\}$  \_\_\_  $\{\{a\}, \{b, c\}\}$
- $0$  \_\_\_  $\emptyset$
- $\emptyset$  \_\_\_  $\emptyset$
- $\emptyset$  \_\_\_  $\{\emptyset\}$
- $\emptyset$  \_\_\_  $\{\{\emptyset\}\}$

# Vzťah „byť prvkom“

Ak je objekt  $x$  prvkom množiny  $A$ , píšeme  $x \in A$ , inak  $x \notin A$ .

- $= \in \{+, -, \cdot, \div, =, (, )\}$
- Star Wars  $\in$  Sci-fi filmy
- $a \notin \{\{a\}, \{b, c\}\}$
- $\{a\} \in \{\{a\}, \{b, c\}\}$
- $0 \notin \emptyset$
- $\emptyset \notin \emptyset$
- $\emptyset \in \{\emptyset\}$
- $\emptyset \notin \{\{\emptyset\}\}$

# Vztah inkluze (podmnožiny)

$$A \subseteq B \stackrel{\text{def.}}{\iff} \forall x: (x \in A \implies x \in B).$$

- Eva      Ženské mená
- $\{1, 2\}$        $\{1, 2, 3\}$
- $\{1, 2\}$        $\{1, 2\}$
- $\emptyset$        $\{1, 2, 3\}$
- $\emptyset$        $\emptyset$

# Vztah inkluze (podmnožiny)

$$A \subseteq B \stackrel{\text{def.}}{\iff} \forall x: (x \in A \implies x \in B).$$

- Eva  $\notin$  Ženské mená
- $\{1, 2\} \subseteq \{1, 2, 3\}$
- $\{1, 2\} \subseteq \{1, 2\}$
- $\emptyset \subseteq \{1, 2, 3\}$
- $\emptyset \subseteq \emptyset$

# Vztah rovnosti

$$A = B \stackrel{\text{def.}}{\iff} \forall x: (x \in A \iff x \in B).$$

- $\{1, 2, 3\}$        $\{3, 1, 2\}$
- $\{1, 2, 3\}$        $\{1, 2, 1, 2, 3, 3, 3, 3\}$
- $\emptyset$        $\emptyset$
- $\emptyset$        $\{\emptyset\}$



# Vztah rovnosti

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$$A = B \stackrel{\text{def.}}{\iff} \forall x: (x \in A \iff x \in B).$$

- $\{1, 2, 3\} = \{3, 1, 2\}$
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- $\emptyset = \emptyset$
- $\emptyset \neq \{\emptyset\}$

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$$\text{Tiež } A = B \stackrel{\text{def.}}{\iff} (A \subseteq B) \wedge (B \subseteq A).$$

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$$\text{Tiež } A = B \stackrel{\text{def.}}{\iff} (A \subseteq B) \wedge (B \subseteq A).$$

$$\text{Podmnožina: } A \subseteq B \stackrel{\text{def.}}{\iff} \forall x: (x \in A \implies x \in B).$$

$$\text{Vlastná podmnožina: } A \subset B \stackrel{\text{def.}}{\iff} A \subseteq B \wedge A \neq B.$$

# Vztah rovnosti

$$A = B \stackrel{\text{def.}}{\iff} \forall x: (x \in A \iff x \in B).$$

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- $\{1, 2, 3\} = \{1, 2, 1, 2, 3, 3, 3, 3\}$
- $\emptyset = \emptyset$
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- $\emptyset \subset \emptyset$

# Vztah rovnosti

$$A = B \stackrel{\text{def.}}{\iff} \forall x: (x \in A \iff x \in B).$$

- $\{1, 2, 3\} = \{3, 1, 2\}$
- $\{1, 2, 3\} = \{1, 2, 1, 2, 3, 3, 3, 3\}$
- $\emptyset = \emptyset$
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$$\text{Tiež } A = B \stackrel{\text{def.}}{\iff} (A \subseteq B) \wedge (B \subseteq A).$$

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$$\text{Vlastná podmnožina: } A \subset B \stackrel{\text{def.}}{\iff} A \subseteq B \wedge A \neq B.$$

- $\emptyset \not\subset \emptyset$

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# Operácia zjednotenia

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$$A \cup B \stackrel{\text{def.}}{=} \{x \mid x \in A \vee x \in B\}.$$

- $\{\text{Adam}\} \cup \{\text{Eva}\} = \underline{\hspace{2cm}}$
- $\{c, b, a\} \cup \{c, b, d\} = \underline{\hspace{2cm}}$
- $\{\{x\}\} \cup \{x\} = \underline{\hspace{2cm}}$
- $\{c, b, a\} \cup \emptyset = \underline{\hspace{2cm}}$
- $\emptyset \cup \emptyset = \underline{\hspace{2cm}}$



# Operácia zjednotenia

---

$$A \cup B \stackrel{\text{def.}}{=} \{x \mid x \in A \vee x \in B\}.$$

- $\{\text{Adam}\} \cup \{\text{Eva}\} = \{\text{Adam}, \text{Eva}\}$
- $\{c, b, a\} \cup \{c, b, d\} = \{a, b, c, d\}$
- $\{\{x\}\} \cup \{x\} = \{x, \{x\}\}$
- $\{c, b, a\} \cup \emptyset = \{a, b, c\}$
- $\emptyset \cup \emptyset = \emptyset$

# Operácia prieniku

$$A \cap B \stackrel{\text{def.}}{=} \{x \mid x \in A \wedge x \in B\}.$$

- $\{\text{Adam}\} \cap \{\text{Eva}\} = \underline{\hspace{2cm}}$
- $\{c, b, a\} \cap \{c, b, d\} = \underline{\hspace{2cm}}$
- $\{\{x\}\} \cap \{x\} = \underline{\hspace{2cm}}$
- $\{c, b, a\} \cap \emptyset = \underline{\hspace{2cm}}$
- $\emptyset \cap \emptyset = \underline{\hspace{2cm}}$

# Operácia prieniku

$$A \cap B \stackrel{\text{def.}}{=} \{x \mid x \in A \wedge x \in B\}.$$

- $\{\text{Adam}\} \cap \{\text{Eva}\} = \emptyset$
- $\{c, b, a\} \cap \{c, b, d\} = \{b, c\}$
- $\{\{x\}\} \cap \{x\} = \emptyset$
- $\{c, b, a\} \cap \emptyset = \emptyset$
- $\emptyset \cap \emptyset = \emptyset$

# Operácia rozdielu

$$A \setminus B \stackrel{\text{def.}}{=} \{x \mid x \in A \wedge x \notin B\}.$$

- $\{\text{Adam}\} \setminus \{\text{Eva}\} = \underline{\hspace{2cm}}$
- $\{c, b, a\} \setminus \{c, b, d\} = \underline{\hspace{2cm}}$
- $\{\{x\}\} \setminus \{x\} = \underline{\hspace{2cm}}$
- $\{c, b, a\} \setminus \emptyset = \underline{\hspace{2cm}}$
- $\emptyset \setminus \emptyset = \underline{\hspace{2cm}}$

# Operácia rozdielu

$$A \setminus B \stackrel{\text{def.}}{=} \{x \mid x \in A \wedge x \notin B\}.$$

- $\{\text{Adam}\} \setminus \{\text{Eva}\} = \{\text{Adam}\}$
- $\{c, b, a\} \setminus \{c, b, d\} = \{a\}$
- $\{\{x\}\} \setminus \{x\} = \{\{x\}\}$
- $\{c, b, a\} \setminus \emptyset = \{c, b, a\}$
- $\emptyset \setminus \emptyset = \emptyset$

# Operácia symetrického rozdielu

$$A \triangle B \stackrel{\text{def.}}{=} (A \setminus B) \cup (B \setminus A).$$

- $\{\text{Adam}\} \triangle \{\text{Eva}\} = \underline{\hspace{2cm}}$
- $\{c, b, a\} \triangle \{c, b, d\} = \underline{\hspace{2cm}}$
- $\{\{x\}\} \triangle \{x\} = \underline{\hspace{2cm}}$
- $\{c, b, a\} \triangle \emptyset = \underline{\hspace{2cm}}$
- $\emptyset \triangle \emptyset = \underline{\hspace{2cm}}$
- $\{1, 5, 9\} \triangle \{1, 5, 9\} = \underline{\hspace{2cm}}$

# Operácia symetrického rozdielu

$$A \triangle B \stackrel{\text{def.}}{=} (A \setminus B) \cup (B \setminus A).$$

- $\{\text{Adam}\} \triangle \{\text{Eva}\} = \{\text{Adam}, \text{Eva}\}$
- $\{c, b, a\} \triangle \{c, b, d\} = \{a, d\}$
- $\{\{x\}\} \triangle \{x\} = \{x, \{x\}\}$
- $\{c, b, a\} \triangle \emptyset = \{c, b, a\}$
- $\emptyset \triangle \emptyset = \emptyset$
- $\{1, 5, 9\} \triangle \{1, 5, 9\} = \emptyset$

# Operácia doplnku

Ak  $A \subseteq U$ , tak doplnok množiny  $A$  vzhľadom k  $U$  je množina  $\bar{A} \stackrel{\text{def.}}{=} U \setminus A$ .

- $U = \{\text{Adam, Barbora, Cyril, Dana}\}$ ,  $A = \{\text{Adam, Cyril}\}$ ,  $\bar{A} = \underline{\hspace{2cm}}$
- $U = \{c, b, a\}$ ,  $\bar{U} = \underline{\hspace{2cm}}$
- $U = \{c, b, a\}$ ,  $\bar{\emptyset} = \underline{\hspace{2cm}}$
- $U = \{1, \{2\}, \{\{3\}\}\}$ ,  $A = \{1\}$ ,  $\bar{A} = \underline{\hspace{2cm}}$



# Operácia doplnku

Ak  $A \subseteq U$ , tak doplnok množiny  $A$  vzhľadom k  $U$  je množina  $\bar{A} \stackrel{\text{def.}}{=} U \setminus A$ .

- $U = \{\text{Adam, Barbora, Cyril, Dana}\}$ ,  $A = \{\text{Adam, Cyril}\}$ ,  $\bar{A} = \{\text{Barbora, Dana}\}$
- $U = \{c, b, a\}$ ,  $\bar{U} = \emptyset$
- $U = \{c, b, a\}$ ,  $\bar{\emptyset} = U$
- $U = \{1, \{2\}, \{\{3\}\}\}$ ,  $A = \{1\}$ ,  $\bar{A} = \{\{2\}, \{\{3\}\}\}$

# Operácia kartézského súčinu

$A \times B \stackrel{\text{def.}}{=} \{(a, b) \mid a \in A \wedge b \in B\}$ .

$(a, b)$  je usporiadaná dvojica definovaná množinou  $\{\{a\}, \{a, b\}\}$ .

- $A = \{1, 2, 3\}$ ,  $M = \{a, b\}$ ,  $A \times M = \underline{\hspace{2cm}}$
- $A = \{1, 2\}$ ,  $B = \emptyset$ ,  $A \times B = \underline{\hspace{2cm}}$
- $A = \{1, 2\}$ ,  $B = \{\emptyset\}$ ,  $A \times B = \underline{\hspace{2cm}}$
- $A = \{x, y\}$ ,  $A^2 = \underline{\hspace{2cm}}$
- $A = \{x, y\}$ ,  $A^0 = \underline{\hspace{2cm}}$

# Operácia kartézského súčinu

$A \times B \stackrel{\text{def.}}{=} \{(a, b) \mid a \in A \wedge b \in B\}$ .

$(a, b)$  je usporiadaná dvojica definovaná množinou  $\{\{a\}, \{a, b\}\}$ .

- $A = \{1, 2, 3\}$ ,  $M = \{a, b\}$ ,  $A \times M = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$
- $A = \{1, 2\}$ ,  $B = \emptyset$ ,  $A \times B = \emptyset$
- $A = \{1, 2\}$ ,  $B = \{\emptyset\}$ ,  $A \times B = \{(1, \emptyset), (2, \emptyset)\}$
- $A = \{x, y\}$ ,  $A^2 = \{(x, x), (x, y), (y, x), (y, y)\}$
- $A = \{x, y\}$ ,  $A^0 = \{\emptyset\}$

# Potenčná množina

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$$2^A \stackrel{\text{def.}}{=} \{B \mid B \subseteq A\}.$$

- $M = \{a, b\}$ ,  $2^M =$
- $2^\emptyset =$

# Potenčná množina

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$$2^A \stackrel{\text{def.}}{=} \{B \mid B \subseteq A\}.$$

- $M = \{a, b\}$ ,  $2^M = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
- $2^\emptyset = \{\emptyset\}$

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